Cosmology in gravity models with broken diffeomorphisms

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- We explore general modifications to the gravitational action that breaks Diff invariance and study the cosmological implications

- Most general global Lorentz invariant action, up to two metric derivatives is

$$S_G = -\frac{1}{16\pi G}\int d^4x \left(\sum_{i=1}^{5} f_i(g)\mathcal{L}_i + f_\Lambda(g)\right)$$

$$\mathcal{L}_1 = -g^{\mu\nu}\Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\nu\alpha}, \quad \mathcal{L}_3 = -g^{\mu\nu}g^{\rho\sigma}g_{\lambda\omega}\Gamma^{\lambda}_{\mu\rho}\Gamma^{\omega}_{\nu\sigma}$$

$$\mathcal{L}_2 = -g^{\mu\nu}\Gamma^{\alpha}_{\mu\nu}\Gamma^{\lambda}_{\lambda\alpha}, \quad \mathcal{L}_4 = -g^{\mu\nu}g^{\rho\sigma}g_{\lambda\omega}\Gamma^{\lambda}_{\mu\nu}\Gamma^{\omega}_{\rho\sigma}$$

$$\mathcal{L}_5 = -g^{\alpha\beta}\Gamma^{\lambda}_{\lambda\alpha}\Gamma^{\mu}_{\mu\beta}$$

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- Einstein-Hilbert action is a particular Diff case of these theories

$$S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{g}\,(\mathcal{L}_2 - \mathcal{L}_1)$$

- We recover Einstein linearized gravity for EH and two additional cases,

  - EH + $\mathcal{L}_4$: $a_1 = -a_2 = -1$, and $a_3 = a_5 = 0$ but $a_4 \neq 0$
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- $\mathcal{L}_5$ is free of ghosts if $a_5 > 0$, and propagates a decoupled scalar graviton

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$$\mathcal{L}_5 = -\frac{1}{4} g^{\mu\nu} (\partial_\mu \ln g)(\partial_\nu \ln g)$$

- Stable model compatible with Newtonian tests

$$S = -\frac{1}{16\pi G}\int d^4x[f_R(g)\, R + f_5(g)\mathcal{L}_5] + \int d^4x\sqrt{g}\,\mathcal{L}_m$$

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$$f_R(g) = \sqrt{g} \rightarrow \gamma_{PPN} = \beta_{PPN} = 1 \quad \forall f_5(g)$$

Enrique Álvarez, Antón F. Faedo, J.J. López-Villarejo (2009) arXiv: 0904.3298

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- For simplicity we consider $f_5(g) = a_5\sqrt{g}$

- To summarize, the action we will use

$$S = -\frac{1}{16\pi G}\int d^4x \left(\sqrt{g}\, R - \frac{a_5\sqrt{g}}{4} g^{\mu\nu}(\partial_\mu \ln g)(\partial_\nu \ln g)\right) + \int d^4x \sqrt{g}\, \mathcal{L}_m$$

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- This action is invariant under tranverse diffeomorphisms (TDiff) $\partial_\mu \xi^\mu = 0$
- Compatible with linearized and post-newtonian GR
- Free of ghosts, instabilities and fifth-force interactions even considering radiative corrections

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- Under solutions of the modified Einstein equations, the TDiff tensor is conserved

$$\nabla_\mu \mathcal{M}^{\mu\nu} = 0$$

Cosmological solutions

- The TDiff piece has the form

$$\mathcal{M}_{\mu\nu} = -\frac{1}{8}(\partial_\alpha \ln g)(\partial_\beta \ln g)(g_{\mu\nu}g^{\alpha\beta} + 2\delta^\alpha_\mu \delta^\beta_\nu) - \frac{1}{2}g_{\mu\nu}\partial_\alpha(g^{\alpha\beta}\partial_\beta \ln g)$$

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- Most general flat FLRW spacetime

$$ds^2 = b^2(\tau)d\tau - a^2(\tau)(dx^2 + dy^2 + dz^2)$$

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$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

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- Friedmann equations are two independent ODE

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{a_5}{3}\mathcal{M}^0_{\ 0} = \frac{8\pi G}{3}\rho$$

$$\frac{\ddot{a}}{a} - \frac{a_5}{6}\left(\mathcal{M}^0_{\ 0} + 3\mathcal{M}\right) = -\frac{4\pi G}{3}(\rho + 3p)$$

- Where $b(\tau)d\tau = dt$ is the cosmological time

- TDiff terms in cosmological time

$$\mathcal{M}^0_{\ 0} = -3\left(\frac{\ddot{a}}{a} + \frac{7}{2}\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{3}\frac{\ddot{b}}{b} - \frac{1}{6}\left(\frac{\dot{b}}{b}\right)^2 + 2\frac{\dot{a}\dot{b}}{ab}\right)$$

$$\mathcal{M} = 3\left(\frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{3}\frac{\ddot{b}}{b} - \frac{1}{2}\left(\frac{\dot{b}}{b}\right)^2\right)$$

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- We end with a two ODE system of H and $H_g = \frac{\dot{g}}{g}$

$$\dot{H} = -\frac{a_5}{8}H_g^2 - 4\pi G(\rho + p)$$

$$\dot{H}_g = -\frac{1}{4}H_g(H_g + 12H) + \frac{6}{a_5}\left(H^2 - \frac{8\pi G}{3}\rho\right)$$

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- Note we need 2 additional parameters, $(H_0, H_{g0}, \Omega_M, \Omega_R, \Omega_\Lambda)$ vs. $(H_0, \Omega_M, \Omega_R)$ in GR.

- Expression for scalar graviton energy

$$\frac{8\pi G}{3}\rho_S = H^2 - \frac{8\pi G}{3}\rho = \frac{a_5}{6}\left(\dot{H}_g + \frac{1}{4}H_g(H_g + 12H)\right)$$

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- This let us to derive an effective equation of state for $\mathcal{L}_5$

$$\omega_S = \frac{p_S}{\rho_S} = -1 + \frac{a_5}{12}\frac{H_g^2}{H^2 - \frac{8\pi G}{3}\rho}$$

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- No phantom or cosmological constant $\rho_S = const.$ behaviour

- Then, the dynamics are related to the ODE system

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- This is equivalent to the ordinary Friedmann equation with an additional fluid with a variable EoS

$$\frac{d\omega_S}{da} = \frac{\omega_S - 1}{a} \left[3(\omega_S + 1) - \mathrm{sgn}(H_g) \sqrt{\frac{8\pi G}{3} \frac{\rho_S}{H^2}} \sqrt{\frac{3}{a_5} (\omega_S + 1)} \right]$$

Explicit solutions

- We first solve simple cases, with **constant $\boldsymbol{\omega_S}$**

$$H^2 = \frac{8\pi G}{3}\rho_s^0 \left(\frac{a}{a_0}\right)^{-3(1+\omega_S)} + \frac{8\pi G}{3}\rho$$

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- Substituing in $\dot{H}, \dot{H}_g$ give us two algebraic equations for H, H_g, ρ_S

$$\rho_S(\omega_S - 1)[H_g + 6(\omega_S + 1)H] = 0$$
$$H_g^2 = \frac{32\pi G\rho_S}{a_5}(\omega_S + 1)$$

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- **Stiff fluid solution.** $\omega_S = 1$, scalar mode behaves as a stiff matter fluid, with $H_g = \pm\sqrt{\frac{24}{a_5}\frac{8\pi G}{3}\rho_S}$

- **Tracker solution.** $H_g + 6(\omega_S + 1)H = 0$. Then ρ and ρ_S fulfills

$$\rho_S(\omega_S - 1)[H_g + 6(\omega_S + 1)H] = 0$$
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- **Vacuum solution.** Tracker sol. with $\rho = 0$, and $\omega_S = \boldsymbol{\omega_S^\infty} \equiv -1 + (3a_5)^{-1}$. **Large a_5 leads to DE**

Approximate solutions

- Equation of evolution for ω_S is integrable in some limits

- **Subdominant case.** $|\rho_S| \ll \rho$, then

$$\frac{d\omega_S}{da} \simeq \frac{3}{a}(\omega_S + 1)(\omega_S - 1) \rightarrow \omega_S = \frac{C - a^6}{C + a^6}$$

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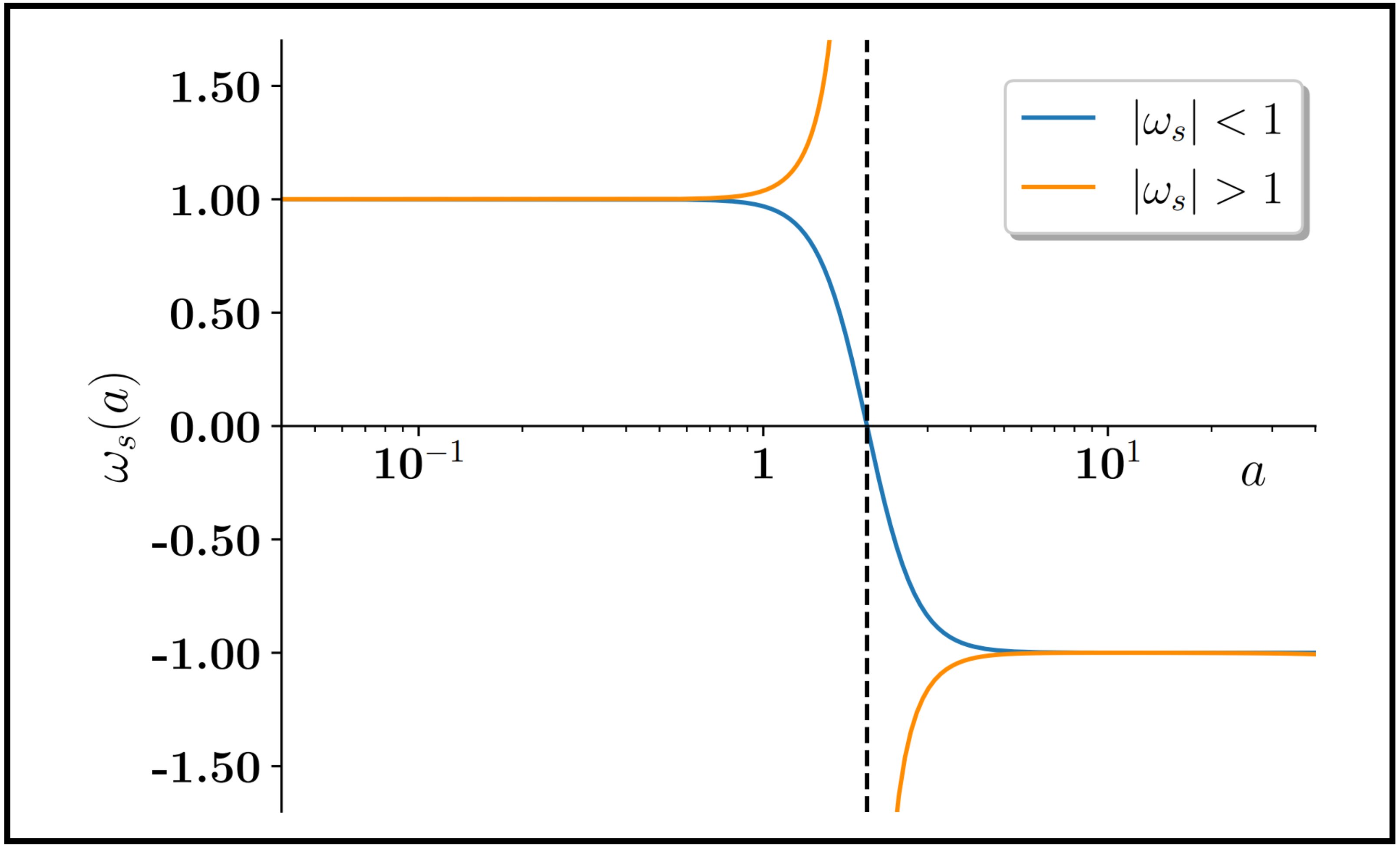
- Transition from early stiff solution to late cosmological constant

- **Subdominant case.** $|\rho_S| \ll \rho$



- **Dominant case.** $|\rho_S| \gg \rho$, the equation reads

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- For $|\omega_S| < 1$
  - $a_5 > \frac{1}{6}$ → Transition between stiff solution and vacuum solution
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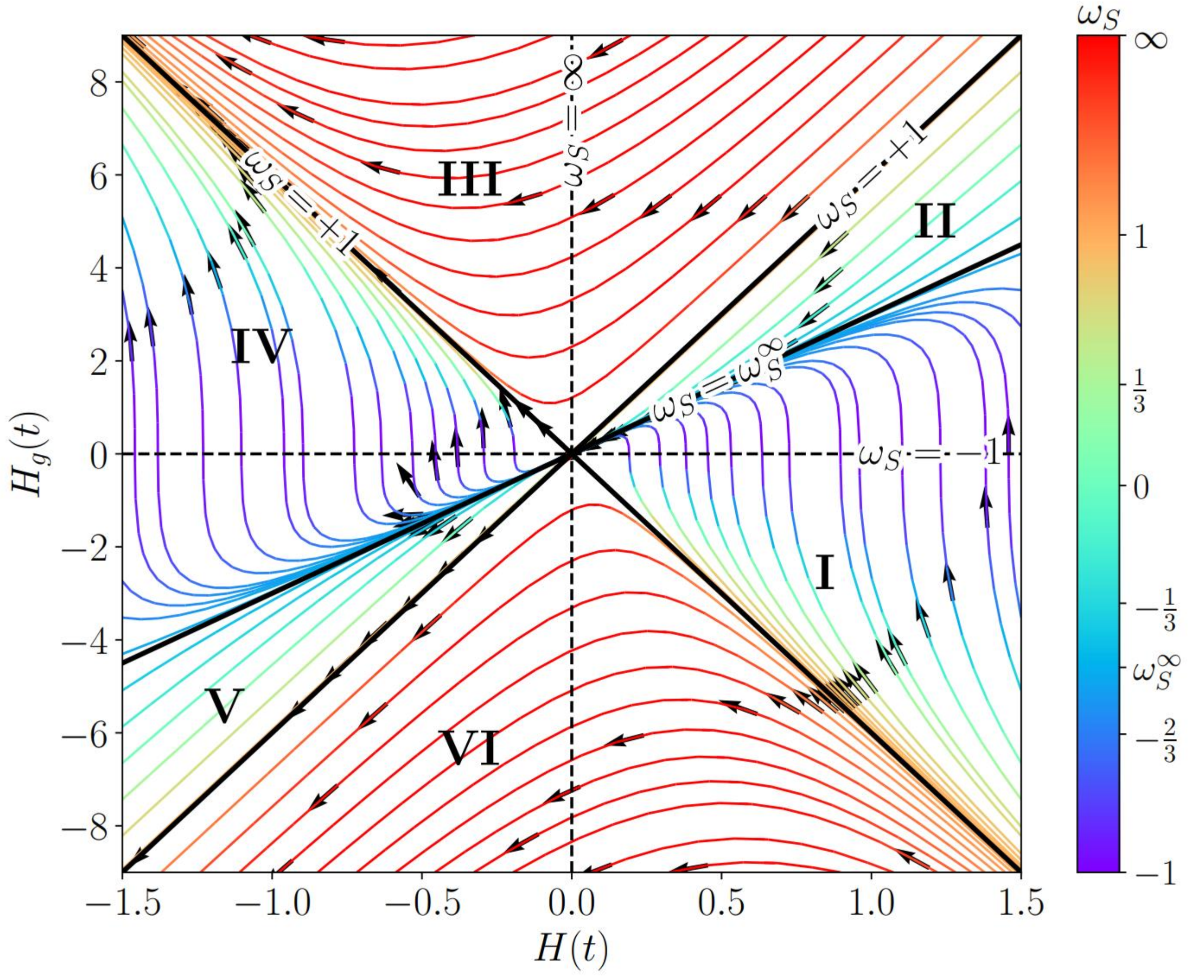
  - $a_5 > \frac{1}{6}$ → Transition between stiff solution and vacuum solution
  - $a_5 < \frac{1}{6}$ → Transition between two stiff solutions
- For $|\omega_S| > 1$, we expect a contraction epoch ($H < 0$)

- **Dominant case.** $|\rho_S| \gg \rho$. Streamline plot of $\dot{H}, \dot{H}_g$

$a_5 > \frac{1}{6}$



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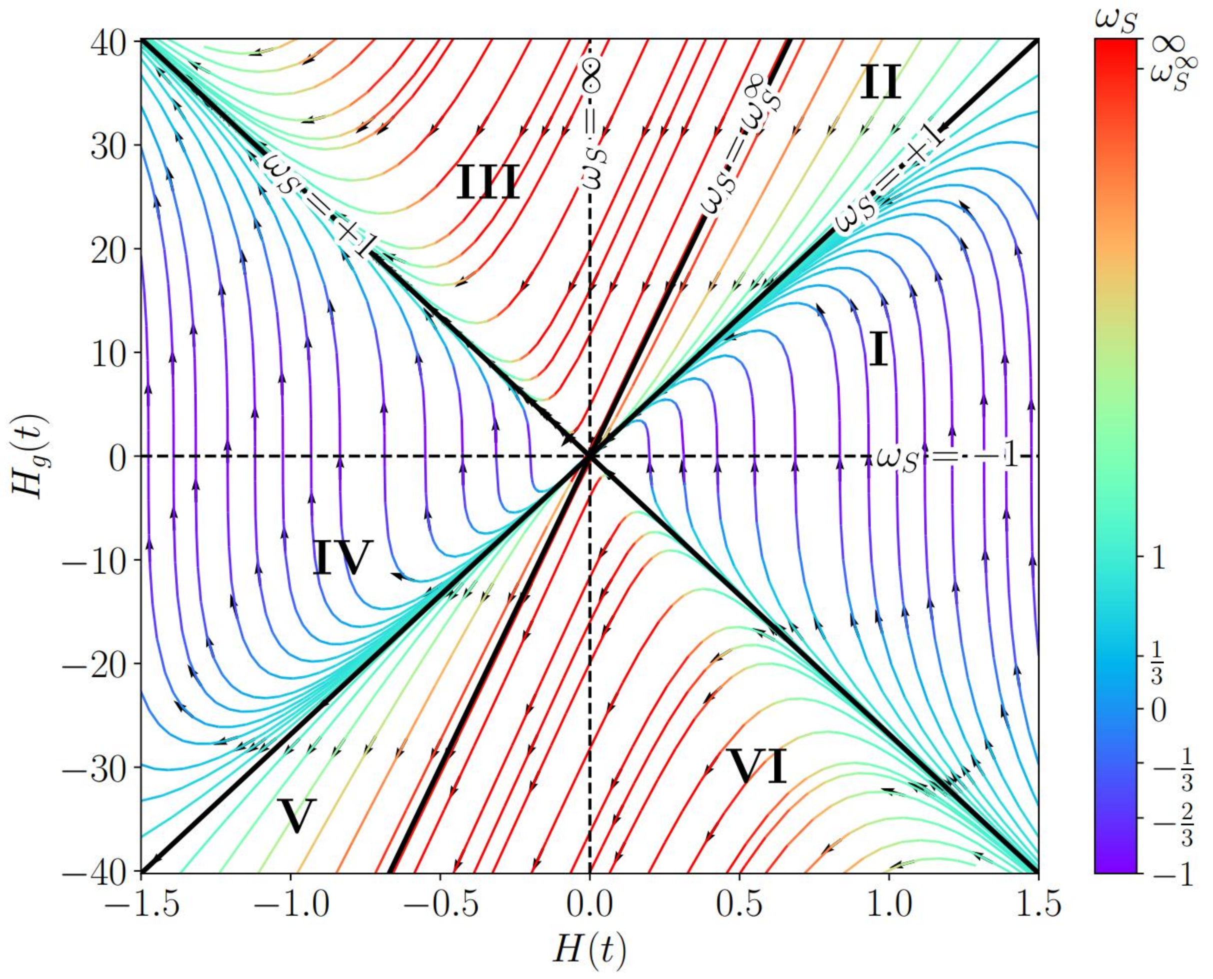

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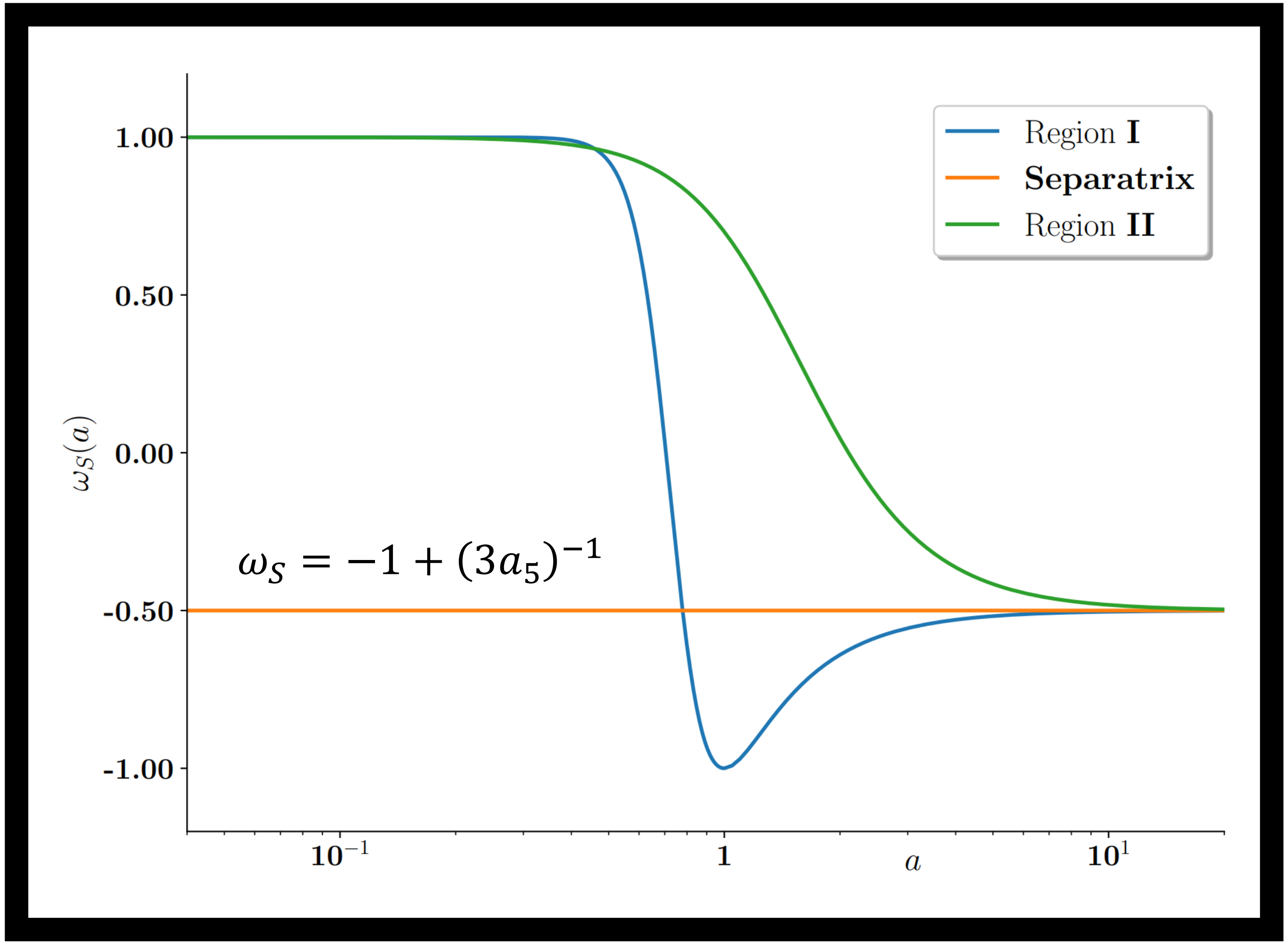
Example for $a_5 > \frac{1}{6}$



General solutions

- Equations for $\dot{H}, \dot{H}_g$ form an autonomous system of 2 + n equations, where n: number of perfect fluids ρ_i

$$\dot{H} = -\frac{a_5}{8} H_g^2 - \frac{3}{2} \sum_{i=1}^{n} H_i^2 (\omega_i + 1)$$

$$\dot{H}_g = -\frac{1}{4} H_g (H_g + 12H) + \frac{6}{a_5} \left(H^2 - \sum_{i=1}^{n} H_i^2 \right)$$

$$\dot{H}_i = -\frac{3}{2} H (\omega_i + 1) H_i, \quad i = 1 \ldots n$$

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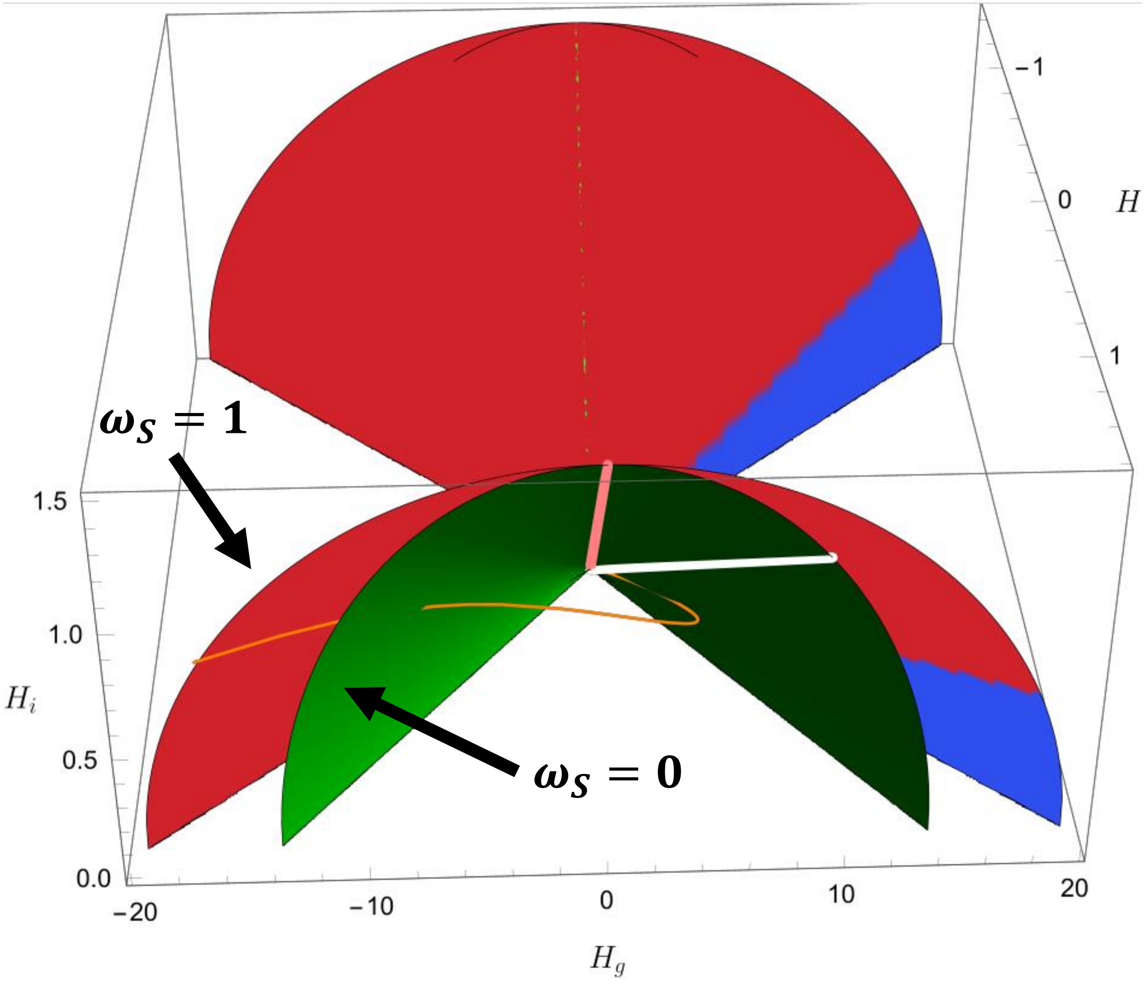
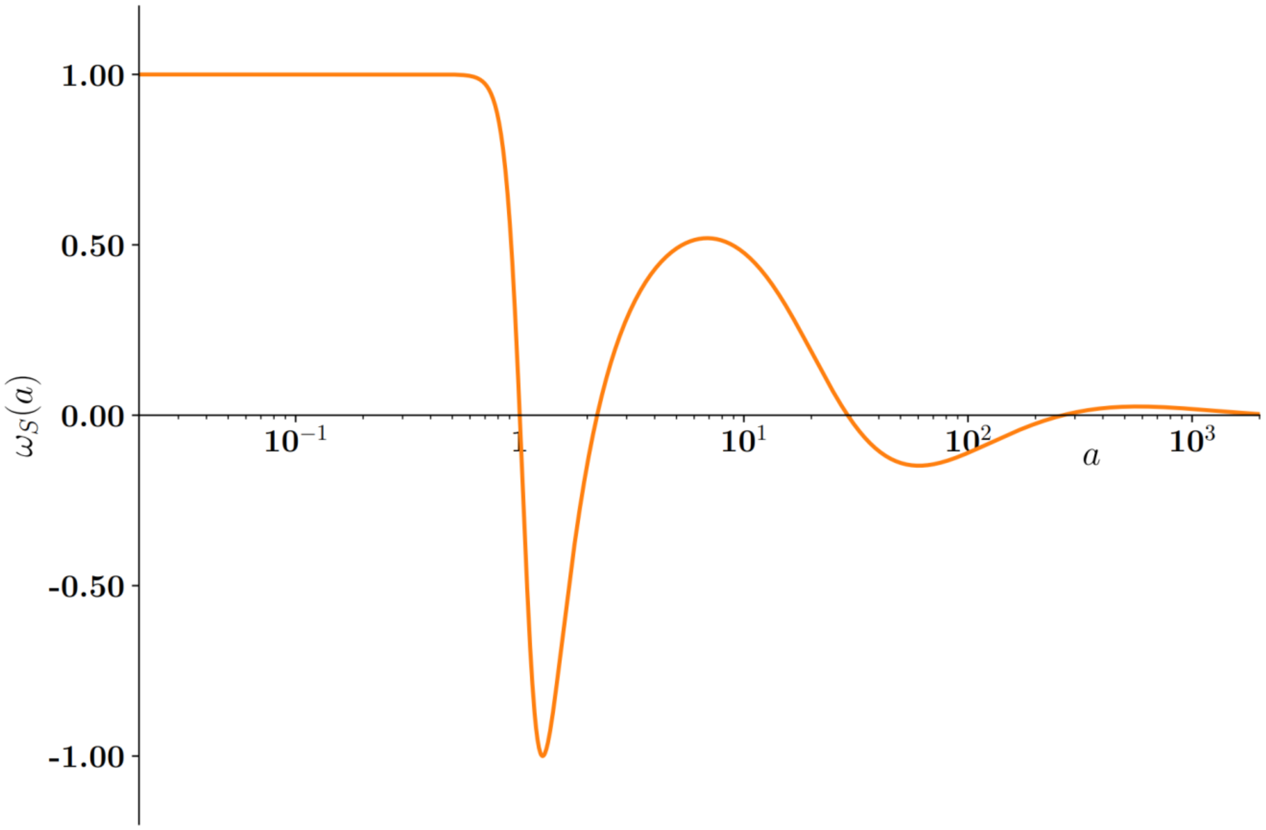
- Solutions asymptotically interpolate between **stiff**, **vacuum** and **tracker** solutions.
- If $\boldsymbol{\omega_S^\infty < \omega_i}$, **vacuum solution at future**, else **tracker**

- **Tracker solution.** $\mathcal{L}_5$ + matter ($\omega = 0$). $a_5 < \frac{1}{6}$





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- Most general metric action up to two metric derivatives, which is stable and compatible with PPN tests.
- The theory propagates an additional scalar graviton and modifies the cosmological evolution
- **ΛCDM** is a particular solution but new solutions are also possible
- Small Diff breaking (ρ_S) freezes as a cosmological constant at late times
- Solutions with large a_5 assymptotically behaves as dark energy. Small a_5 lead to tracker solutions

Thank you for your attention

$$\mathcal{L}_{int} = \kappa_1^2 \left[T^*_{\mu\nu} T^{\mu\nu} - \frac{1}{(n-2)} |T|^2 \right] \frac{1}{k^2} - \left(\kappa_2 + \frac{1-a}{n-2} \kappa_1 \right)^2 \frac{|T|^2}{\Delta b\ k^2 - \ m_2^2}.$$

E. Álvarez, D. Blas,
J. Garriga, E. Verdaguer (2006)
arXiv: hep-th / 0606019